No. of Printed Pages : **2**

PJ-275

100220

I Semester M.Sc. Examination, February - 2020
(CBCS - Y2K17/Y2K14 Scheme)

MATHEMATICS

M104T : Ordinary Differential Equations

Time : 3 Hours

Max. Marks : 70

Instructions : *(i) Answer **any five** full questions.*
*(ii) **All** questions carry **equal** marks.*

1. (a) Prove that $W\{\phi_j(x),\ j=1 \text{ to } n\} = W\{\phi_j(x_0),\ j=1 \text{ to } n\}\exp\left\{-\int_{x_0}^{x} a_1(t)/a_0(t)\,dt\right\}$, where $\{\phi_j(x),\ j=1 \text{ to } n\}$ are n solutions of $L_n y=0$ on I and $x,\ x_0 \in I$. Discuss any two deductions from this result. **10**

(b) Verify Lagrange's identity for $x^2y''-3xy'+3y=0$. **4**

2. (a) Establish that $L_n^{**}=L_n$. **5**

(b) Explain the method of variation of parameters of solving $L_n y=b(x)$. **9**

3. (a) State and prove Sturm's comparison theorem on the zeros of $\{p(x)y'\}'+q(x)y=0$ on [a, b] and illustrate it with an example. **9**

(b) Is the existence and uniqueness theorem is valid for the IVP : $y'=5y^{4/5}$; $y(0)=0$ in the domain $|x|\le 1$ and $|y|\le 1$? Discuss it. **5**

4. (a) Obtain the eigen function expansion formula and hence expand e^x in terms of orthogonal eigen functions of $y''+\lambda y=0;\ y(0)=y(\pi)(\lambda > 0)$. **9**

(b) Outline Green's method of solving non-homogeneous eigen value problem. **5**

5. (a) Using the Frobenius method obtain the Laguerre polynomial from the differential equation $xy''+(1-x)y'+\alpha y=0$. **8**

(b) Derive the Rodrigue's formula for Laguerre polynomials. **6**

P.T.O.

(a) Prove the orthogonality of the Chebyshev polynomial of the first kind. 8

(b) $\frac{1-t^2}{1-2tx+t^2}$ is the generating function of $T_n(x)$. Prove this statement. 6

7. For the system $\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ find the fundamental matrix and the general solution. Find e^{tA} for the coefficient matrix A in the above system. 14

8. (a) Find the nature of the critical point of the system : $\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+2y+x\cos y \\ -y-\sin y \end{bmatrix}$. 7

(b) Given the system : $\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x+2x^2+2y^2 \\ -y+xy \end{bmatrix}$, determine the stability of its critical point (0, 0) by constructing the Liapunov function. 7

- o 0 o -